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# Reconstruction in space and visualization of a planar image: a mathematical and computational introduction

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**Abstract.** Aim of this paper is to provide a short introduction to the fundamental mathematical ideas involved in tri-dimensional (3D) image reconstruction, in a basic way as to make such ideas available to a large audience. Particular attention is then given to the case in which the bi-dimensional (2D) input data are represented of very few images. A feasibility study is outlined for a 3D reconstruction of a human gland or organ, satisfying hypothesis of *uniformity* in the tissue and in the arterial distribution, spacial extension and density. The resulting method aims at obtaining functional information on the target organ, minimizing the number (and social cost) of possibly invasive imaging techniques by employing one or very few *scintigraphic or echographic* images. (www.actabiomedica.it)

**Key words:** Scintigraphy, edge detection, data approximation, parametrization in space

## Introduction

Humans are visual creatures. We categorize visual features important to us, comparing the size of an object with that of a standard, or relating its shape with other familiar shapes. Image analysis attempts to formalize this type of process. As Alvarez and Morel say in (1):

"Image processing might be viewed as a long list of techniques for capturing, transmitting and extracting information from digital images, in close relation with what is assumed to be relevant to human perception".

Analyzing an image comprises various steps and components, that may be grouped as follows:

- image acquisition devices;
- means for converting the image to digital form;
- software, as well as hardware, to process the data in order to extract the desired information.

Here we focus on the third step, in particular, summarizing the mathematical and computational concepts that arise in software devoted to solving problems of spacial reconstruction of a planar image. This overview aims to reach the widest possible audience and, as such, it is described in a very simplified way. For a thorough reference we refer again to (1).

## Digital imaging

Few technologies have offered as much potential to research and teaching as *digital imaging*, not just in scientific and technological fields, but also in the arts and humanities.

A digital image is composed of a set of dots or squares, called *pixels* (from *picture elements*), arranged in a grid (*matrix*) of columns and rows. A number corresponds to each pixel, representing the brightness level (color or shade of gray) of that pixel in the image.

There are different ways to obtain a digital image. Either the image is digital already from the beginning or it has to be digitized in some way, which means that we need to decide which color each pixel should be assigned. The digital matrix is created during a scanning acquisition process: the analog original is sampled, usually at regular intervals, or the color of selected points of its surface is recorded. The combination of all neighboring pixels creates the illusion of a continuous image. Generally speaking, the more samples taken from the source image, the more accurate the (*resolution* of the) resulting digital replacement will be.

## Medical images

In Medicine, images are for example provided by *tomography*, that is to say slice-wise images of a 3D volume, for instance the human body. One way to obtain tomographic images is to use X-ray: this is called *computerized tomography* (CT); each image corresponds to one slice of the body and is directly stored in the computer. If we compare a tomographic image to a regular X-ray image, we see that all structures are put on top of each other: it is a *projection*. With CT we make many projections, at different angles around the body, from which we can reconstruct the 3D volume.

Other ways to construct such images is to use *magnetic resonance tomography* (MR). In an MR image, the gray-level corresponds to the amount of hydrogen that there is in the material at each position. With so called *positron emission tomography* (PET), furthermore, we obtain an image showing the amount of activity in each position.

*Ultrasonography* is a technique in which high-frequency sound waves are bounced off internal organs, proportionally to the different densities; the echo pattern is converted into a 2D picture, in which each intensity (gray level) is characteristic of each organ or tissue. An ultrasound is a non invasive technique, useful in determining morphology and size of an organ, but it is highly operator-dependent and its application in obtaining functioning information is not very straightforward.

Another imaging study is *scintigraphy*, whose visual information give indication on the organ functioning. Nuclear medicine procedures consist of the administration of a radio-pharmaceutical with special affinity for the organ or tissue of interest.

In the study of the thyroid, for example, radioactive iodine or technetium are used by injection. They accumulate in the target organ after a certain time and decay emitting an essentially mono-energetic *gamma ray* (photons with energies greater than 100 kilo-electron Volts). The activity (distribution of the radioactivity) is detected using a *gamma camera* and images are displayed on the computer. Areas in which there is more accumulation are called *hot* and show higher functioning; vice-versa regions in which there is little or no accumulation are called *cold* and show low or no functioning. Nuclear medicine images are usually of poor resolution, but provide unique information about specific organs. Scintigraphic imaging, in particular, is very useful in the diagnosis of the thyroid Plummer's disease (Figure 1).

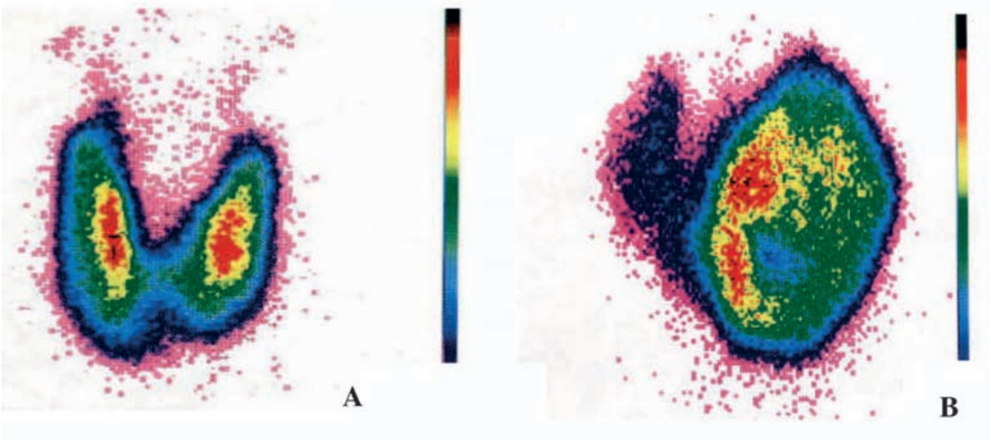

Figure **1.** Scintigraphic images: normal thyroid **(left)**; Plummer's disease **(right)**

*Synthetic images*

These are images that we create ourselves in the computer, for testing solution methods: with synthetic images it is easier to foresee what the result of a specific method should be.

## Preprocessing

Once we have the image in our computer, it is often necessary to improve it, before starting with the actual analysis of it. Different kinds of disturbances (*noise*) in the image may be present, due, for instance, to errors in the sensor collecting the data.

Or we may want to use different features in the image, like edges between regions, and would like to *enhance* those.

The *portrait of Lena* is well known in the world of computers: it is often used as a test image, as there are many different gray levels in it and many different types of texture (that is to say, *patterns;* see for instance the plume). Compare the original image in Figure 2a with that in Figure 2b, in which the so called *salt & pepper* noise has been added: there are a lot of pixels that, for some reason, are white. To improve the degraded image, we scan through it and look at each pixel and its closest neighbors: if the pixel is very different from its neighbors, we change it to a value which fits better in the neighborhood. This can be done in many different ways, using different *thresholds* to vary the pixel values: a result is shown in Figure 2c.

*Edge detection*

To find edges in an image is often of interest. An edge means a fast transition from dark to light or the other way around. One example on how an image can look like when the edges have been detected can be seen in Figure 2d. The dark indicates that the pixels in that region have similar gray levels. Fast transitions (edges) are shown in white.

Being a jump in intensity, the cross section of an edge has the shape of a ramp: an ideal edge is a discontinuity, that is a ramp with an infinite slope; the first derivative assumes a local maximum at an edge, so we can make use of this to localize edges (2).

These concepts can easily be extended to the case of a continuous 2D image: the two *directional* first order partial derivatives are used to form a function called gradient magnitude, whose local maxima identify the image edges. Discrete *finite difference* approximations of the directional derivatives are used and the discrete analogous of the gradient magnitude is defined and referred to as *gradient image*.

A pixel location is declared an edge location if its value of the gradient image exceeds some threshold; the locations of all edge points constitutes an *edge map*.

## Problem modeling and solution

Given a single 2D ultrasonogram (echographic image) or scintigraphic image for one patient, we consider the feasibility for a 3D reconstruction of a hu-

Figure **2.** Images of Lena (*left to right*): original **(a)**; noisy **(b)**; preprocessed **(c)**; edges **(d)**.

man gland or organ. In particular, the reconstruction of one lobe of the thyroid gland is described in detail in (3).

Hypothesis of *uniformity* in the tissue and in the arterial distribution, extension and density are made.

These assumptions can be reasonably made in the case of a thyroid lobe and nodule. They also apply to other organs or glands in the human body, such as liver, kidneys, lungs, suprarenal glands, hypophysis, ovaries, testicles, mammary glands, etc.

The input data to our problem is thus represented by the scan of a 2D scintigraphic image; colors are rendered as gray levels by means of the formula *gray = 0.299 red + 0.587 green + 0.114 blue*. The output sought, besides the visualization of the organ in space, is the approximation of the functional/morphologic volume of the organ: this value is also taken as the approximation of the vascularization volume (4). From the numerical point of view of computational mathematics, the approximated volume has to fulfill criteria of accuracy, while the implementation of the reconstruction method must be robust and efficient in terms of memory requirements, computational time and cost. The solving algorithm can be outlined as follows:

**i: edge map** - the gray level 2D image is stored as a matrix, to which an edge detection method (plus some refining technique) is applied in order to determine the border of the organ; Figure 3a illustrates the case of one thyroid lobe;

**ii: approximation** - the edge points are collected into two sets in such a way that each set can be approximated by a function; the organ border is thus split into two silhouettes **s1** and **s2**: the thyroid example is shown in Figure 3b; the analytic form of **s1** and **s2** can be recovered by numerical data approximation in the polynomial *least squares* sense: let **s** be a real continuous function of one real variable, sampled at **m** distinct abscissas **xi**; we determine the polynomial **p**, of chosen degree **n < m**, that minimizes the (sum of the squares of the) errors **p(xi) – s(xi)**;

**iii: parametrization** - a parametrization in space is applied in order for **s1** to rotate and, at the same time, be smoothly deformed into **s2.** Figure 4 illustrates the idea by employing two synthetic silhouettes. Note that a pure rotation in space would not guarantee a closed 3D object, as Figure 5 shows;

**iv: volume** - with the correct parametrization, a 3D object is obtained (Figure 6), with known analytic expression, that can therefore be integrated to obtain the volume value (5).

## Conclusions

For more than hundred years, radiography has given insight into our phenotype, permitting disease detection based on the gross impact of the disease upon body morphology, that is to say at the scale of the organism.

The advent of tomographic techniques, including CT, MR and ultrasound, has allowed visualization of morphologic perturbations at the scale of the organ.

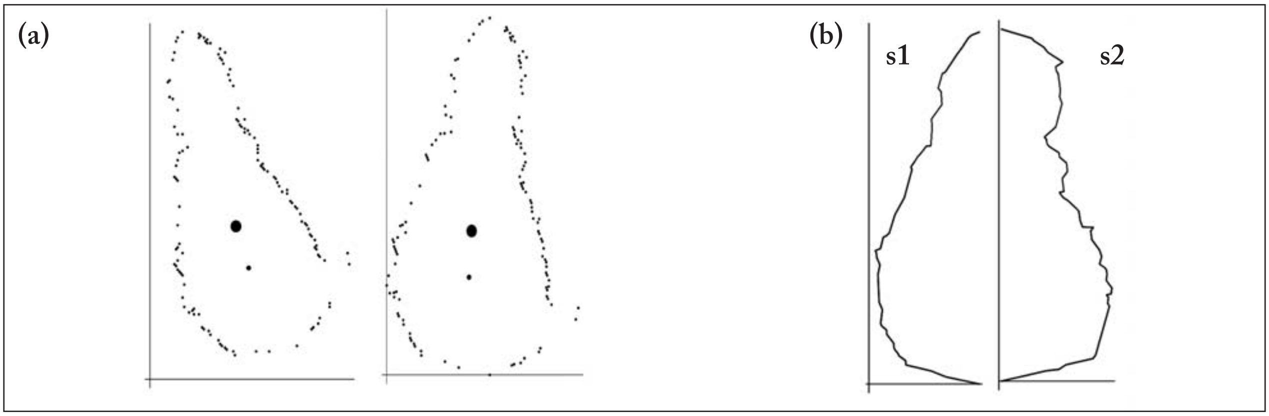

**Figure 3.** *From left to right*: border points of one lobe of a thyroid, before and after rotation with respect to a vertical symmetry axis **(a)**; their approximation as two separate sets **(b)**, s1 and s2

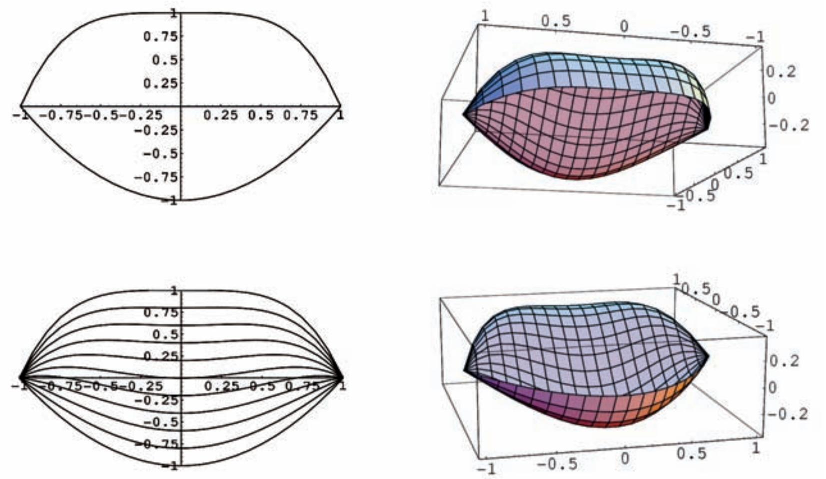

Figure **4.** Correct result obtained by the space parametrization

An alternative strategy to improving early detection is to search for functional changes. Historically, nuclear medicine gave insight into our functional health; for example, iodine uptake has long been used to diagnose thyroid disease. While such methods were performed at the scale of the organ, the sensitivity of such methods to early transformations of the phenotype was poor: diseases were most commonly detected only after the organ morphology was altered sufficiently so as to result in gross functional changes. Advances in radioactive agents, the advent of PET and the introduction of contrast agents and functional imaging techniques to MR have dramatically advanced this field. Today, there are numerous *functional imaging* methods. Such methods now routinely allow detection of lesions which would be missed with morphologic methods.

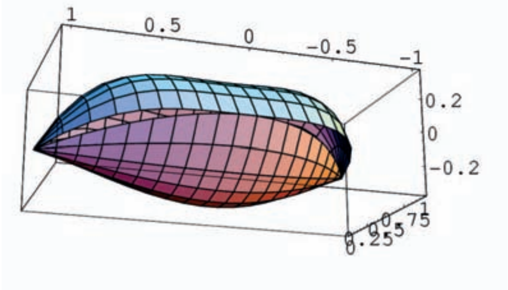

Figure **5.**Wrong result obtained by pure revolution

The method presented here relies heavily on the uniformity assumption previously stated and aims at obtaining prompt and accurate functional information on the organ, minimizing at the same time the number (and social cost) of possibly invasive imaging techniques, since it makes use of data from one planar scintigraphy or from very few ultrasound images.

The stability features of the method presented in this paper are essentially those of polynomial approximation. The computational cost also coincides mainly with that of the data approximation. Three

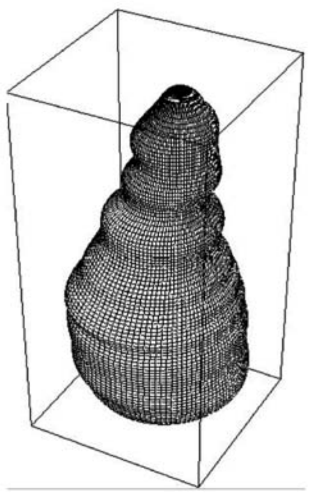

Figure **6.** Volume visualization after space parametrization matrices, all of the same size, are required: the first matrix stores the reference image values and two more matrices are needed in the edge detection procedure. The method application to a 256 x 128 image, for example, requires a few seconds (elapsed time) on a personal computer, having an 800 MHz processor and 128 MB of RAM memory, mounting the Red Hat Linux operating system and running the *Mathematica* software.

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